Tukey’s Honestly Significant Difference (HSD)

Let $y_1, y_2, \ldots, y_t$ be independent observations from $N(\mu, \sigma^2)$ and $w = \max(y_i) - \min(y_i)$, and $s^2$ be the estimate of $\sigma^2$ which is based on $\nu$ degrees of freedom, then $q(t, \nu) = w/s$ is called the studentized range (Table A.10, Ott). When $n_i = n, n_T = nt$,

$$\bar{y}_i - \bar{y}_j \approx N \left(0, \frac{s^2}{n} \right)$$

$$\frac{MSE}{n} = \frac{s^2}{n}$$

The population means $\mu_i$ and $\mu_j$ are significantly different if

$$|\bar{y}_i - \bar{y}_j| \geq q_{a, (t, \nu)} \sqrt{\frac{MSE}{n}}$$

Tukey’s Honestly Significant Difference (HSD) (Tukey-Kramer procedure)

The $(1-\alpha)100\%$ Tukey’s confidence interval estimate of $\mu_i - \mu_j$ is

$$\bar{y}_i - \bar{y}_j \pm q_{a, (t, \nu)} \sqrt{\frac{MSE}{n}}$$

The population means $\mu_i$ and $\mu_j$ are significantly different if

$$|\bar{y}_i - \bar{y}_j| \geq q_{a, (t, \nu)} \sqrt{\frac{MSE}{n}}$$

Fisher’s LSD : $t_{a/2, (t-1)} \sqrt{\frac{MSE}{n}} = 2.064 \sqrt{\frac{2451}{5}} = 64.63$

Tukey’s HSD : $q_{a, (t, \nu)} \sqrt{\frac{MSE}{n}} = 4.37 \sqrt{\frac{2451}{5}} = 96.75$

Tukey’s procedure is more conservative than the LSD procedure and would declare fewer significant difference.