A company wish to compare the performance four different tires (A, B, C, and D). There are four cars available for this study. It is believe that tires wearing out in a different rate at different location of a car. Tires are installed in four different locations: Right-Front, Left-Front, Right-Rear and Left-Rear.

- **Your Tasks**: Design the study. You must take into account what type of design you would use, data collection, assignment of tires to cars, and analysis.

Latin Square Design (Incomplete Block Design)

- Suppose in an experiment that we have two blocking factors. If there are r treatment levels and both block factors has r levels, then we would need r^3 experimental units.
- A r x r Latin Square Design will reduce the number experiment units to r^2.

r x r Latin Square Design

It is a design that contain r rows and r columns. The r treatments are randomly assigned to experimental units within the rows and columns so that each treatment appears in every row and in every column (exactly once).

	RF	LF	RR	LR
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

- **Standard Latin Squares** are the squares in which the elements of the first row and the first column are arranged alphabetically.
- Randomization procedure: select a standard square at random and arrange its rows and columns at random.

- Advantages:
  - Appropriate for comparing r treatment means in presence of two sources of extraneous variation each has r levels.
  - Simple

- Disadvantage:
  - It is better for $5 \le r \le 10$.
  - Any additional extraneous source of variability tend to inflat the error and make it difficult to detect difference among means.
  - The effect of treatment on the response must be approximately the same across the rows and columns.
  - Interaction can not be considered.

Data from studying tires

Positions

Cars		RF	LF	RR	LR
	1	A(32)	B(33)	C(47)	D(53)
	2	B(36)	D(53)	A(42)	C(54)
	3	C(51)	A(44)	D(62)	B(49)
	4	D(81)	C(78)	B(72)	A(73)

Statistical Model

$$Y_{ij(k)} = \mu + \rho_i + \kappa_j + \tau_k + \varepsilon_{ij(k)}$$

μ : overall mean

ρ_i : main effect of the i-th row

κ_j : main effect of the j-th column

τ_k : main effect of the k-th treatment

$\varepsilon_{ij(k)} \sim$ *i.i.d Normal*$(0, \sigma^2)$

$\Sigma \rho_i = \Sigma \kappa_j = \Sigma \tau_k = 0$

Latin Square ANOVA Summary Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
Row	**r - 1**	**SSR**	**MSR**	**MSR / MSE**
Column	**r – 1**	**SSC**	**MSC**	**MSC / MSE**
Treatment	**r – 1**	**SST**	**MST**	**MST / MSE**
Error	**(r-1)(r-2)**	**SSE**	**MSE**	
Total	**r^2 - 1**	**SS(Total)** ← **Corrected SSTotal**		

ANOVA Table for Tire Study

Tests of Between-Subjects Effects

Dependent Variable: TIREWEAR

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Model	49854.500[a]	10	4985.450	19941.800	.000
CAR	2850.500	3	950.167	3800.667	.000
POSITION	133.500	3	44.500	178.000	.000
TIRE	645.500	3	215.167	860.667	.000
Error	1.500	6	.250		
Total	49856.000	16			

a. R Squared = 1.000 (Adjusted R Squared = 1.000)