Analysis of Categorical Data

Goodness of Fit Test

(Examine Distribution)

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Example:

The color distribution of A&A candies is supposed to be 30% red, 20% green, and 50% yellow. In a random sample of 200 A&A candies taken from the production line, 56 red, 52 green and 92 yellow. Does the sample suggest that the distribution of different color of candies still follows the 30/20/50 distribution?

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Chi-square Goodness of Fit Test (For Distribution of Univariate Data)

The Chi-square Goodness-of-fit Test (Pearson, 1900)

$F(x)$: the true but unknown distribution function.

$F^*(x)$: a completely specified distribution function.

H_0: $F(x) = F^*(x)$ for all x

H_a: $F(x) \neq F^*(x)$ for at least one x

	Classes				
	1	2	...	k	***Total***
Observed frequencies	O_1	O_2	...	O_k	***N***
Probability distribution from $F^*(x)$	π_1^*	π_2^*		π_k^*	**1**
Expected frequencies	E_1	E_2	...	E_k	***N***

(Expected Cell Count: $E_j = \pi_j^* N$ $j = 1, 2, ..., k$)

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Test Statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2\{k - 1\}$$

Under H_0, χ^2 has a chi-square distribution with $(k-1)$ degrees of freedom

Cochran's guidelines: (Large sample conditions)

- None of the expected cell counts less than 1.
- No more than 20% of the expected cell frequencies are less than 5.

Decision Rule:

Reject H_0 If $\chi^2 > \chi^2_\alpha$ or ***p*-value** $< \alpha$.

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Example: (continue)

	Red	Green	Yellow	Total
Observed Frequencies	56	52	92	200
Expected Frequencies	60	40	100	200

$$\chi^2 = \frac{(56-60)^2}{60} + \frac{(52-40)^2}{40} + \frac{(92-100)^2}{100}$$

$= 4.507 < 5.991 = \chi^2_{.05}(2)$

=> fail to reject H_0

Conclusion: There is no sufficient evidence to support that the color distribution is significantly different from 30/20/50.

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Example:

It is believe that the outcomes is uniformly distributed from a game of drawing a ball from 4 balls numbered from 1, 2, 3, and 4 in a box. The outcomes from the past 100 games were recorded, and the frequency distribution (count) is the following: 1 occurred 20 times, 2 occurred 25 times, 3 occurred 41 times and 4 occurred 14 times. Does the sample suggest that the outcome distribution of this game is not significantly different from uniform distribution?

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Test of Homogeneity and Independence in a Two-way Table

(Examine Correlation between Two Categorical Variables)

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Is there a relationship between Treatment and Heart Disease?

Heart Disease Variable:
"Have the disease" or "Do not have the disease."
Treatment Variable:
"Placebo" or "Aspirin".

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36	114	150
Aspirin	14	136	150
Total	50	250	300

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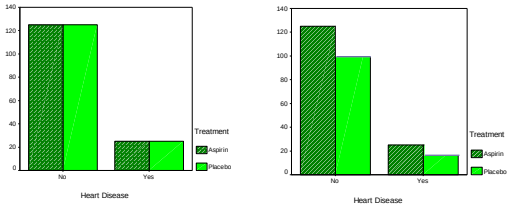
Evidence of No Correlation

Cluster bar chart



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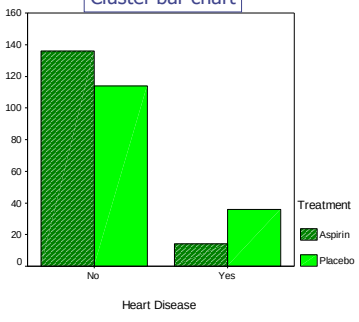
Evidence of Correlation

(as in the example)

Cluster bar chart



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Questions about the sample

- Did this happen by chance?
- How likely would it happen if there is no significant treatment effect?

Testing statistical hypotheses!

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Null hypothesis: There is **no relationship** between treatment variable and outcome variables.

Alternative hypothesis: There **is relationship** between treatment variable and outcome variables

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36 (25)	114 (125)	150
Aspirin	14 (25)	136 (125)	150
Total	50	250	300

$\frac{150 \times 50}{300} = 25$

Observed frequency

Expected frequency

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Expected Frequency:

Numbers in (..) :
(i,j)th cell expected freq. $= \dfrac{M_i \times N_j}{T}$

M_i: i-th **row** total
N_j: j-th **column** total
T : grand total

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Chi-square Test for Independence

Test statistic:

$\chi^2 = (36 - 25)^2/25 + (14 - 25)^2/25$
$+ (114 - 125)^2/125 + (136 - 125)^2/125 =$ **11.616**

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36 (25)	114 (125)	150
Aspirin	14 (25)	136 (125)	150
Total	50	250	300

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Test Statistic:

$$\chi^2 = \sum_{i=1}^{rc} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2\{(r-1)(c-1)\}$$

Under H_0, χ^2 has a chi-square distribution with $(r-1)(c-1)$ degrees of freedom

Cochran's guidelines: (Large sample conditions)
- None of the expected cell counts less than 1.
- No more than 20% of the expected cell frequencies are less than 5.

Decision Rule:

Reject H_0 If $\chi^2 > \chi^2_\alpha$ or ***p*-value** $< \alpha$.

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Decision Rule

- *p*-value approach:
  Reject H_0 if *p*-value $< .05$
- Critical Value approach:
  Reject H_0 if the test statistic $> \chi^2_\alpha = 3.84$

TABLE A.8
Percentiles of the chi-square distribution

	Area in Upper Tail				
df	0.100	0.050	0.025	0.010	0.001
1	2.71	3.84	5.02	6.63	10.83
2	4.61	5.99	7.38	9.21	13.82
3	6.25	7.81	9.35	11.34	16.27

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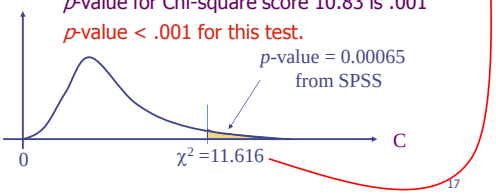
p-value

	Area in Upper Tail				
df	0.100	0.050	0.025	0.010	0.001
1	2.71	3.84	5.02	6.63	10.83
2	4.61	5.99	7.38	9.21	13.82
3	6.25	7.81	9.35	11.34	16.27

The probability of observing an evidence (statistic) that is at least as extreme as the observed sample evidence, if the null hypothesis is true.

p-value for Chi-square score 10.83 is .001
p-value < .001 for this test.



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Decision (Conclusion)

Reject H_0 since *p*-value $< .001 < 0.05$ and $\chi^2 =$ **11.616** $> \chi^2_\alpha =$ **3.84**.

There is sufficient evidence to support the alternative hypothesis that there is statistically significant association between treatment and heart disease.

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Relative Risk

Relative risk of getting heart disease for taking Placebo versus Aspirin is (36/150) / (14/150) = 2.57

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36	114	150
Aspirin	14	136	150
Total	50	250	300

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Step 1: Test the hypothesis at 5% level of sig.

Hypothesis:

H_0: There is **NO** relation between variable 1 (treatment) and variable 2 (outcome variables).

H_A: There is relation between two variables.

	Relapse		
	No	Yes	Total
Desipramine	14 (8)	10 (16)	24
Lithium	6 (8)	18 (16)	24
Placebo	4 (8)	20 (16)	24
Total	24	48	72

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Step 2:

Test Statistics: $\chi^2 = \frac{(14-8)^2}{8} + \frac{(10-16)^2}{16} + \frac{(6-8)^2}{8} + \frac{(18-16)^2}{16} + \frac{(4-8)^2}{8} + \frac{(20-16)^2}{16} = \mathbf{10.5}$

Step 3:

d.f.= (3–1)(2–1) = 2

C.V. approach: Reject null hypothesis if $\chi^2 > \chi^2_{.05} = 5.99$

p-value approach: Reject null hypothesis if the ***p*-value** < α.

Step 4:

Conclusion: Since $\chi^2 = 10.5 > \chi^2_{.05} = 5.99$ or $\chi^2 = 10.5 > 9.21$, the ***p*-value** < 0.01 < 0.05. The relation between treatment and outcome variables is statistically significant.

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TABLE A.8

Percentiles of the chi-square distribution

	Area in Upper Tail				
df	0.100	0.050	0.025	0.010	0.001
1	2.71	3.84	5.02	6.63	10.83
2	4.61	5.99	7.38	9.21 **10.5**	13.82
3	6.25	7.81	9.35	11.34	16.27

0.001 < *p*-value < .01

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Example: Two independent random samples from two countries were collected.
Country A: 551 of 1500 adults were smokers.
Country B: 652 of 2000 adults were smokers.
At α = 0.05, test whether there is a significant difference between the percentages of smokers in country A and country B?

	Smoke	Not Smoke	Total
A	551	949	1500
B	652	1348	2000
Total	1203	2297	3500

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Cautions

- Confounding factors: race, age, …
- Randomization, Design of experiment.
- Use of regression or multiple contingency tables.

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