Exercise Problems on One Sample Statistical Inference

1. A researcher wishes to estimate the average heart performance score (HPS) for a certain population. A sample of 9 subjects randomly chosen from this population was taken and data is the following:

   Data: 6.5, 5.5, 7.1, 4.5, 5.6, 5.1, 6.8, 6.6, 6.3

   Mean = 6.0; S.D. = 0.87

   a) Find the 95% confidence interval estimate for the average heart performance score for the sampled population.

   \[ \bar{x} \pm t_{\frac{\alpha}{2}, df=n-1} \frac{s}{\sqrt{n}} \Rightarrow 6.0 \pm 2.306 \frac{0.87}{\sqrt{9}} \]

   \[ 6.0 \pm 0.67 \]

   \[ (5.33, 6.67) \]

   b) If one wishes to estimate the average HPS for this population with a margin of error within 0.2, how large a sample would be needed? (The standard deviation is believed to be around 1.2 and is to be used for this sample size estimation.)

   \[ n = \left( \frac{Z_{\alpha} \cdot \sigma}{E} \right)^2 \]

   \[ = \left( \frac{1.96 \cdot 1.2}{0.2} \right)^2 = 139.29 \approx 139 \]
2. In a study of health behavior, a random sample of 300 subjects from a major city was surveyed. In this random sample, 170 responded that they had dinner mostly after 8 PM.

a) Use this data to construct a 95% confidence interval to estimate the percentage of people in the sampled population who had dinner mostly after 8 PM.

\[
\hat{p} = \frac{x}{n} = \frac{170}{300} = 0.5667
\]

\[
\hat{p} \pm Z_{0.025} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.5667 \pm 1.96 \cdot \sqrt{\frac{0.5667 \cdot 0.4333}{300}}
\]

\[
= 0.5667 \pm 0.0581
\]

\[
56.67\% \pm 5.81\%
\]

b) If one wishes to estimate the percentage of people in the sampled population who had dinner mostly after 8 PM with margin of error no more than 5%, how large a sample is required?

\[
h = \left( \frac{Z_{0.025}}{E} \right)^2 \cdot 0.25
\]

\[
= \left( \frac{1.96}{0.05} \right)^2 \cdot 0.25
\]

\[
= 384.16 \rightarrow 385
\]

c) If one wishes to estimate the percentage of people in the sampled population who had dinner mostly after 8 PM with margin of error no more than 5%, how large a sample is required? (It is believed this percentage is around 60% and will be used for the sample size estimation.)

\[
h = \left( \frac{Z_{0.025}}{E} \right)^2 \cdot \hat{p} \cdot (1-\hat{p})
\]

\[
= \left( \frac{1.96}{0.05} \right)^2 \cdot 0.6 \cdot 0.4
\]

\[
= 368.79 \rightarrow 369
Exercise Problems on One Sample Statistical Inference

3. A researcher hypothesized that the average heart performance score (HPS) for a certain population is less than 7. A sample of 9 subjects randomly chosen from this population was taken and data is the following:

Data: 6.5, 5.5, 7.1, 4.5, 5.6, 5.1, 6.8, 6.6, 6.3

Mean = 6.0; S.D. = 0.87

a) What is the proper test procedure for this analysis?

one sample t-test

c) Check the normality assumption using statistical software output of normality test and comment on the normality assumption using p-value.

\[ p\text{-value} = 0.6375 \Rightarrow 0.05 = \alpha \]

We do not reject normality assumption

d) State the hypothesis.

Null hypothesis: \( \mu = 7 \)

Alternative hypothesis: \( \mu < 7 \)

\( \alpha = 0.05 \)

\( \alpha = 5\% \)

\( \text{df} = n-1 = 9-1 = 8 \)

\( t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{6 - 7}{\frac{0.87}{\sqrt{9}}} = -3.45 \)

\( t_{0.05, 8} = 1.885 \)

f) What is the decision rule? (Use either p-value approach.)

Reject \( H_0 \) if \( p\text{-value} < \alpha \).

\( p\text{-value} = 0.004294 \)

RTT: \( p\text{-value} = 1 - 0.004294 \)

g) Draw a conclusion for this test using p-value approach.

\( 0.004 < p\text{-value} < 0.05 \Rightarrow \text{reject } H_0 \)

There is sufficient evidence to support \( H_0 \), that the average HPS is less than 7, at 5\% level of significance.
Exercise Problems on One Sample Statistical Inference

4. A researcher wishes to perform a one sample test for the **average weight reduction** to see if the new treatment can **reduce** the weight using a **level of significance** of $\alpha = .05$.

   a) Find the sample size so that one can have a 90% power to reject the null hypothesis if the new treatment can actually help reducing weight by **10 lbs** on average. (The standard deviation of weight reduction measures is approximately equal 30.)

   $$n = \frac{(Z_{\alpha}+Z_{\beta})^2 \cdot \sigma^2}{\mu_a - \mu_0} = \frac{(1.645 + 1.282) \cdot 30^2}{10}$$

   $$\frac{90}{100} = 1 - \beta$$

   $$Z_{.05} = 1.645$$

   $$Z_{.1} = 1.282$$

   $$\beta = .1$$

   $$n = 77.07 \approx 78$$

   $$\Rightarrow$$

   b) Find the sample size so that one can have a 90% power to reject the null hypothesis if the new treatment can actually help reducing weight by **10 lbs** on average with a two-sided test. (The standard deviation of weight reduction measures is approximately equal 30.)

   $$n = \frac{(Z_{\alpha/2}+Z_{\beta/2}) \cdot \sigma^2}{\mu_a - \mu_0} = \frac{(1.96 + 1.282) \cdot 30^2}{10}$$

   $$Z_{.025} = 1.96$$

   $$Z_{.1} = 1.282$$

   $$n = 42.03 \approx 43$$