Tests of Statistical Hypotheses

1 Tests about Proportions

Inference on Proportion

Parameter: Population Proportion p (or π)

(Percentage of people has no health insurance)

Statistic: Sample Proportion $\hat{p} = \frac{x}{n}$

x is number of successes

n is sample size

Data: 1, 0, 1, 0, 0 $\Rightarrow \hat{p} = \frac{2}{5} = .4 \quad \Rightarrow \hat{p} = \bar{x}$

$\bar{x} = \frac{1+0+1+0+0}{5} = .4$

Sampling Distribution of Sample Proportion

A random sample of size n from a large population with proportion of successes (usually represented by a value 1) p, and therefore proportion of failures (usually represented by a value 0) $1 - p$, the sampling distribution of **sample proportion**, $\hat{p} = x/n$, where x is the number of successes in the sample, is **asymptotically normal** with a **mean** p and **standard deviation** $\sqrt{\frac{p(1-p)}{n}}$.

Confidence Interval

Confidence interval: The $(1-\alpha)$x100% confidence interval estimate for population proportion is

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Large Sample Assumption:

Both np and $n(1-p)$ are greater than **5**, that is, it is ***expected that there at least 5 counts in each category***.

An Alternative Method

By solving for $\frac{y}{n} = \hat{p}$ in $\frac{|y/n - p|}{\sqrt{p(1-p)/n}} \le z_{\alpha/2}$

The $(1-\alpha)$x100% Confidence Interval for p is

$$\frac{\hat{p} + z_{\alpha/2}^2/(2n) \pm z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})/n + z_{\alpha/2}^2/(4n)^2}}{1 + z_{\alpha/2}^2/n}$$

Hypothesis Testing

1. State research hypotheses or questions. $p = 30\%$?
2. Gather data or evidence (observational or experimental) to answer the question. $\hat{p} = .25 = 25\%$
3. Summarize data and test the hypothesis.
4. Draw a conclusion.

Statistical Hypothesis

Null hypothesis **(H_0)**:

Hypothesis of no difference or no relation, often has =, ≥, or ≤ notation when testing value of parameters.

Example:

H_0: $p = 30\%$ or

H_0: Percentage of votes for A is 30%.

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Statistical Hypothesis

Alternative hypothesis (H_1 or H_a)

Usually corresponds to research hypothesis and opposite to null hypothesis,

often has >, < or ≠ notation in testing mean.

Example:

H_a: $p \neq 30\%$ or

H_a: Percentage of votes for A is not 30%.

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Hypotheses Statements Example

- A researcher is interested in finding out whether percentage of people in favor of policy A is different from 60%.

H_0: $p = 60\%$

H_a: $p \neq 60\%$

[Two-tailed test]

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Hypotheses Statements Example

- A researcher is interested in finding out whether percentage of people in a community that has health insurance is more than 77%.

H_0: $p = 77\%$ (or $p \leq 77\%$)

H_a: $p > 77\%$

[Right-tailed test]

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Hypotheses Statements Example

- A researcher is interested in finding out whether the percentage of bad product is less than 10%.

H_0: $p = 10\%$ (or $p \geq 10\%$)

H_a: $p < 10\%$

[Left-tailed test]

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Evidence

Test Statistic **(Evidence)**: A sample statistic used to decide whether to reject the null hypothesis.

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Logic Behind Hypothesis Testing

In testing statistical hypothesis,

***the null hypothesis is first assumed to be true**.*

We collect evidence to see if the evidence is strong enough to reject the null hypothesis and support the alternative hypothesis.

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One Sample Z-Test for Proportion (Large sample test)

Two-Sided Test

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I. Hypothesis

One wishes to test whether the percentage of votes for A is different from 30%

H_o: p = 30% v.s. H_a: $p \neq$ 30%

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Evidence

What will be the key statistic (evidence) to use for testing the hypothesis about population proportion?

Sample Proportion: p

A random sample of 100 subjects is chosen and the sample proportion is 25% or .25.

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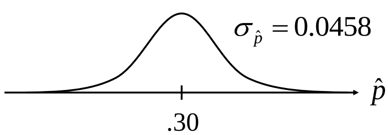
Sampling Distribution

If **H_0:** p = 30% **is true**, sampling distribution of sample proportion will be approximately normally distributed with *mean* **.3** and *standard deviation* (or standard error) $\sqrt{\frac{.3 \cdot (1-.3)}{100}} = 0.0458$

$\sigma_{\hat{p}} = 0.0458$



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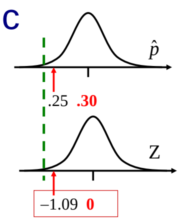
II. Test Statistic

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1-p_0)}{n}}}$$

$$= \frac{.25 - .3}{\sqrt{\frac{.3 \cdot (1-.3)}{100}}} = -1.09$$



This implies that the statistic is 1.09 standard deviations away from the mean .3 under H_0 , and is to the left of .3 (or less than .3)

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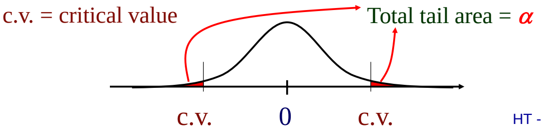
Level of Significance

***Level of significance* for the test (α)**

A probability level selected by the researcher at the beginning of the analysis that defines unlikely values of sample statistic if null hypothesis is true.



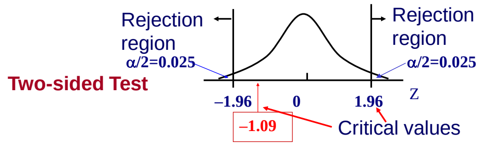
III. Decision Rule

Critical value approach: Compare the test statistic with the critical values defined by significance level α, usually α = 0.05.

We reject the null hypothesis, if the test statistic

$z < -z_{\alpha/2} = -z_{0.025} = -1.96$, or $z > z_{\alpha/2} = z_{0.025} = 1.96$.

(i.e., $|z| > z_{\alpha/2}$)

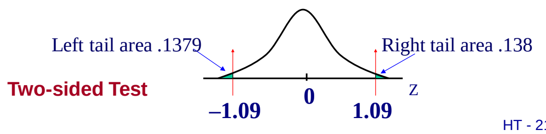


III. Decision Rule

***p*-value approach:** Compare the probability of the evidence or more extreme evidence to occur when null hypothesis is true. If this probability is less than the level of significance of the test, α, then we reject the null hypothesis. (Reject H_0 if *p*-value < *α*)

p-value = $P(Z \le -1.09 \text{ or } Z \ge 1.09)$

= 2 x $P(Z \le -1.09)$ = 2 x .1379 = **.2758**



p-value

♥ ***p-value*** ♥

The probability of obtaining a test statistic that is as extreme or more extreme than actual sample statistic value given null hypothesis is true. It is a probability that indicates the extremeness of evidence against H_0.

The smaller the p-value, the stronger the evidence for supporting Ha and rejecting H_0.

IV. Draw conclusion

Since from either
critical value approach $z = -1.09 > -z_{\alpha/2} = -1.96$
or p-value approach p-value = .2758 > α = .05 ,
we do not reject null hypothesis.

Therefore we conclude that **there is no sufficient evidence to support the alternative hypothesis that the percentage of votes would be different from 30%.**

Steps in Hypothesis Testing

1. State hypotheses: H_0 and H_a.
2. Choose a proper **test statistic**, collect data, checking the assumption and compute the value of the statistic.
3. Make decision rule based on **level of significance(α)**.
4. Draw conclusion.
   (Reject or not reject null hypothesis)
   (Support or not support alternative hypothesis)

When do we use this z-test for testing the proportion of a population?

- Large random sample.

One-Sided Test

Example with the same data:
A random sample of **100** subjects is chosen and the sample proportion is **25%** .

I. Hypothesis

One wishes to test whether the percentage of votes for A is less than 30%

H_o: p = 30% v.s. H_a: p < 30%

Evidence

What will be the key statistic (evidence) to use for testing the hypothesis about population proportion?

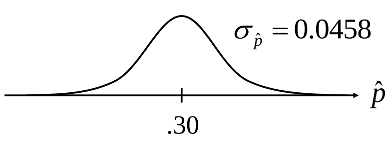
Sample Proportion: $\hat{p}$

A random sample of 100 subjects is chosen and the sample proportion is 25% or .25.

Sampling Distribution

If **H_0:** p = 30% **is true**, sampling distribution of sample proportion will be approximately normally distributed with *mean* **.3** and *standard deviation* (or standard error) $\sqrt{\frac{.3\cdot(1-.3)}{100}} = 0.0458$

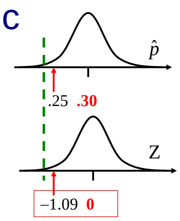
$\sigma_{\hat{p}} = 0.0458$



II. Test Statistic

$$z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0\cdot(1-p_0)}{n}}}$$

$$= \frac{.25 - .3}{\sqrt{\frac{.3\cdot(1-.3)}{100}}} = -1.09$$



This implies that the statistic is 1.09 standard deviations away from the mean .3 under H_0 , and is to the left of .3 (or less than .3)

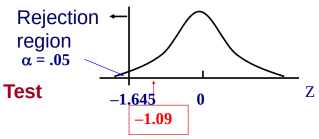
III. Decision Rule

Critical value approach: Compare the test statistic with the critical values defined by significance level α, usually $\alpha = 0.05$.

We reject the null hypothesis, if the test statistic

$z < -z_\alpha = -z_{0.05} = -1.645$,

Left-sided Test

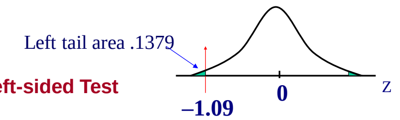


III. Decision Rule

***p*-value approach:** Compare the probability of the evidence or more extreme evidence to occur when null hypothesis is true. If this probability is less than the level of significance of the test, α, then we reject the null hypothesis.

p-value = $P(Z \le -1.09) = P(Z \le -1.09) =$ **.1379**

Left-sided Test



Z-Table

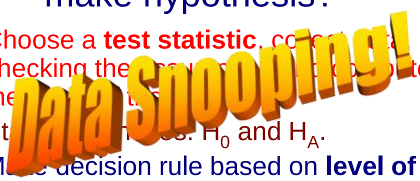
IV. Draw conclusion

Since from either
critical value approach $z = -1.09 > -z_{\alpha/2} = -1.645$
or p-value approach p-value = .1379 > α = .05 ,
we do not reject null hypothesis.

Therefore we conclude that **there is no sufficient evidence to support the alternative hypothesis that the percentage of votes is less than 30%.**

Can we see data and then make hypothesis?

1. Choose a **test statistic**, co… checking the … te the …
2. St… H_0 and H_A.
3. M… decision rule based on **level of significance(α)**.
4. Draw conclusion. (Reject null hypothesis or not)

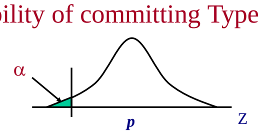


Errors in Hypothesis Testing

Possible statistical errors:

- ***Type I error***: The null hypothesis is true, but we reject it.
- ***Type II error***: The null hypothesis is false, but we don't reject it.

"α" is the probability of committing Type I Error.



One-Sample *z*-test for a population proportion

z-test:

Step 1: State Hypotheses (choose one of the three hypotheses below)

i) $H_0: p = p_0$ v.s. $H_A: p \ne p_0$ (Two-sided test)
ii) $H_0: p = p_0$ v.s. $H_A: p > p_0$ (Right-sided test)
iii) $H_0: p = p_0$ v.s. $H_A: p < p_0$ (Left-sided test)

Hypothesis Testing for Proportions

Test Statistic

Step 2: Compute z test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

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Step 3: Decision Rule:

***p*-value approach:** Compute p-value,

if $H_A: p \neq p_0$, p-value $= 2 \cdot P(Z \geq |z|)$

if $H_A: p > p_0$, p-value $= P(Z \geq z)$

if $H_A: p < p_0$, p-value $= P(Z \leq z)$

reject H_0 if p-value $< \alpha$

Critical value approach: Determine critical value(s) using α, reject H_0 against

i) $H_A: p \neq p_0$, if $|z| > z_{\alpha/2}$

ii) $H_A: p > p_0$, if $z > z_\alpha$

iii) $H_A: p < p_0$, if $z < -z_\alpha$

Step 4: Draw Conclusion.

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Example: A researcher hypothesized that the percentage of the people living in a community who has no insurance coverage during the past 12 months is **not 10%**. In his study, 1000 individuals from the community were randomly surveyed and checked whether they were covered by any health insurance during the 12 months. Among them, 122 answered that they did not have any health insurance coverage during the last 12 months. Test the researcher's hypothesis at the level of significance of 0.05.

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Hypothesis: $H_0: p = .10$ v.s. $H_A: p \neq .10$ (Two-sided test)

Test Statistic: $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{.122 - .10}{\sqrt{\frac{.10(1-.10)}{1000}}} = 2.32$

p-value = 2 x .0102 = .0204

Decision Rule: Reject null hypothesis if p-value < .05.

Conclusion: p-value = .0204 < .05. There is sufficient evidence to support the alternative hypothesis that the percentage is statistically significantly different from 10%.

Ex. 8.10

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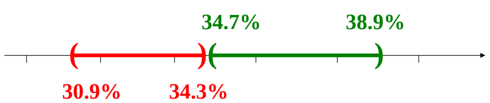
Confidence Interval Estimate of One Proportion

$\hat{p}_1 = 551/1500 = .367 = 36.7\%$ (from A)
$\hat{p}_2 = 652/2000 = .326 = 32.6\%$ (from B)

For A: 36.7% ± 2% or (34.7%, 38.9%)
For B: 32.6% ± 1.7% or (30.9%, 34.3%)



Two CI's do not overlap implies significant difference.

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Methods of Testing Hypotheses

- Traditional Critical Value Method
- P-value Method
- Confidence Interval Method

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