Introduction to Estimation

Confidence Intervals
&
Sample Size

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Disadvantage of Point Estimation

1. Provides Single Value
   Based on Observations from 1 Sample.
   * Sample Mean $\bar{X} = 98$ Is a Point Estimate of Unknown Population Mean.
2. Gives No Information about How Close Value Is to the Unknown Population Parameter

Which of the following statistics do you prefer?
a. 32%
b. 32% with a margin of error 3%

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Estimation

You're interested in finding the average body temperature of healthy adults in Northeastern Ohio (the population). What would you do?

How can we estimate this average with a measure of reliability?

98 ± 1 F° | 98 ± .5 F° | 98 ± .2 F°

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Interval Estimation

Margin of Error Gives Information about How Close Value Is to the Unknown Population Parameter.

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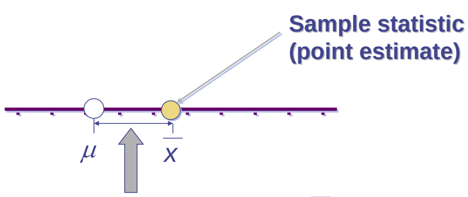
Sampling Error



Sampling Error = $|\mu - \bar{x}|$

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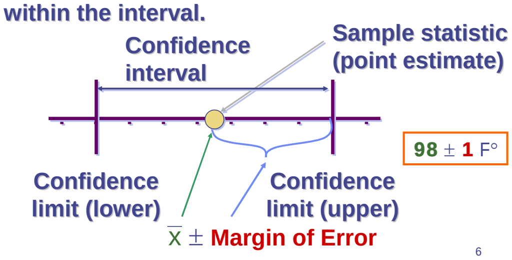
Key Elements of Interval Estimation

<u>Confidence Level</u>: A probability that the population parameter falls somewhere within the interval.



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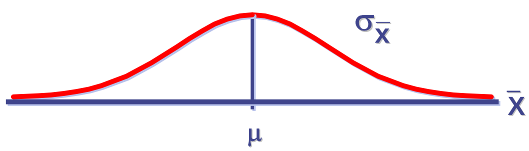
Sampling Distribution of the Mean



The sampling distribution is normal when **sampled from normally distributed population** or **having a relatively large sample**.

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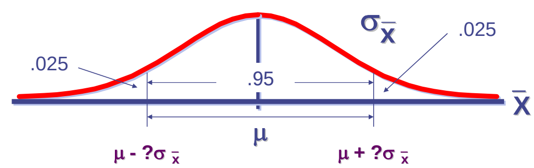
Sampling Distribution of the Mean



Within how many standard deviations of the mean will have 95% of the sampling distribution?

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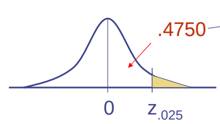
A Special Notation

z_α = the z score that the proportion of the standard normal distribution to the right of it is α.

$z_{.025} = 1.96$

$z_{.010} = ?$



Z	.05	.06	.07
1.8	.4678	.4686	.4693
1.9	.4744	.4750	.4756
2.0	.4798	.4803	.4808
2.1	.4842	.4846	.4850

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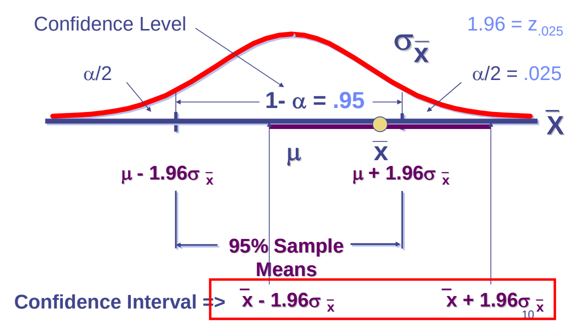
The Confidence Interval



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Confidence Interval for Mean (σ Known)

- (1-α)·100% Confidence Interval Estimate for mean of a **normal** population

$$\left(\bar{X} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \, , \, \bar{X} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

or

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Margin of Error

"σ Known" may mean that we have very good estimate of σ.
It is not practical to assume that we know σ.

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Confidence Interval of Mean (σ unKnown and n ≥ 30)

- (1-α)·100% Confidence Interval Estimate for mean of a population when sample size is relative large

$$\left(\bar{X} - Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \, , \, \bar{X} + Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \right)$$

or

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

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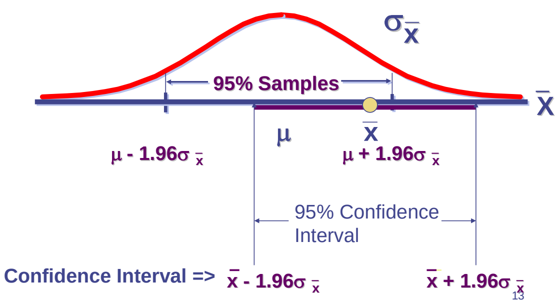
The Confidence Interval



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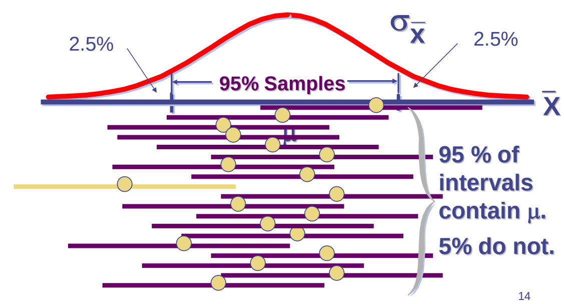
The Confidence Interval



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Factors Affecting Interval Width

$$\left(\bar{X} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \;,\; \bar{X} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

1. Data Dispersion
   Measured by σ
2. Sample Size
   Affects standard error: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
3. Level of Confidence $(1 - \alpha)$
   Affects $Z_{\alpha/2}$

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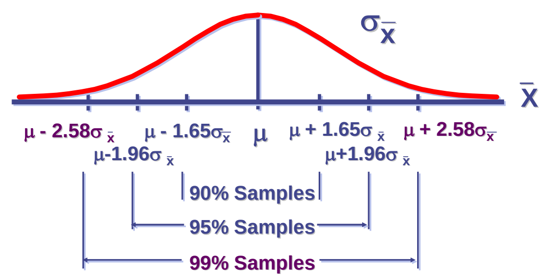
Size of Interval



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Thinking Challenge

- You're a Q/C inspector for Coca Cola. A random sample of **100** bottles showed **X = 1.99** liters and **s** = 0.**05** liters. What is the **90%** confidence interval estimate of the true **mean** amount in 2-liter bottles?

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Confidence Interval Solution*

$$1-\alpha = .90,\ \alpha = 1-.90 = .1,\ \alpha/2 = .05$$

$$Z_{\alpha/2} = Z_{.05} = 1.64$$

$$\bar{X} \pm Z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

$$1.99 \pm 1.64 \cdot \frac{.05}{\sqrt{100}} \Rightarrow 1.99 \pm 0.008$$

$$\Rightarrow (\ 1.982\,,\ 1.998\ )$$

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Interpretation

◈We can be 95% confident that the population mean is in (1.982, 1.998).

◈We can be 95% confident that the maximum sampling error using 1.99 for estimating the population mean is within 0.008.

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Thinking Challenge

Example: A city uses a certain noise index to monitor the noise pollution at a certain area of the city. A random sample of **100** observations from randomly selected days around noon showed an **average index** value of **$\bar{x}$ = 1.99** and **standard deviation s = 0.05**. Find the **90%** confidence interval estimate of the **average** noise index at noon.

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Confidence Interval Solution*

$$1-\alpha = .90,\ \ \alpha = 1-.90 = .1,\ \ \alpha/2 = .05$$

$$Z_{\alpha/2} = Z_{.05} = 1.64$$

$$\bar{X} \pm Z_{\alpha/2}\cdot\frac{s}{\sqrt{n}}$$

$$1.99 \pm 1.64\cdot\frac{.05}{\sqrt{100}} \Rightarrow 1.99 \pm 0.008$$

$$\Rightarrow (\ 1.982\ ,\ 1.998\ )$$

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Finding Sample Sizes for Estimating μ

C.I.: $\bar{x} \pm z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$

Margin of Error $= B = Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$

$$n = \frac{z^2_{\alpha/2}\cdot\sigma^2}{B^2}$$

***B* = Margin of Error or Bound**

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Sample Size Example

What sample size is needed to be 90% confident of being correct within ± 5? A pilot study suggested that the standard deviation is 45.

$$n = \frac{Z_{.05}{}^2\sigma^2}{B^2} = \frac{(1.645)^2(45)^2}{(5)^2} = 219.2 \cong 220$$

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Thinking Challenge

◈You plan to survey residents in your county to find the average health insurance premium that they are paying. You want to be **95%** confident that the sample **mean** is within ± **$50**.
A pilot study showed that σ was about **$400**. What **sample size** should you use?

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Sample Size Solution*

$$n = \frac{Z_{0.025}^{2}\sigma^2}{B^2}$$

$$= \frac{(1.96)^2(400)^2}{(50)^2}$$

$$= 245.86 \cong 246$$

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Confidence Interval of Mean (σ unKnown and n ≥ 30)

- $(1-\alpha)\cdot 100\%$ Confidence Interval Estimate for mean of a population when sample size is relative large

$$\left(\bar{X} - Z_{\alpha/2}\cdot\frac{s}{\sqrt{n}} \;,\; \bar{X} + Z_{\alpha/2}\cdot\frac{s}{\sqrt{n}} \right)$$

or

Review

$$\bar{X} \pm Z_{\alpha/2}\cdot\frac{s}{\sqrt{n}}$$

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Confidence Interval Mean (σ Unknown & n< 30)

1. Assumptions
   Population Standard Deviation Is Unknown
   Population Must Be **Normally Distributed**
2. Use Student's t Distribution
3. Confidence Interval Estimate

$$\left(\bar{X} - t_{\alpha/2, n-1}\cdot\frac{S}{\sqrt{n}} ,\; \bar{X} + t_{\alpha/2, n-1}\cdot\frac{S}{\sqrt{n}} \right)$$

$$\bar{X} \pm t_{\alpha/2,\, n-1}\cdot\frac{S}{\sqrt{n}}$$

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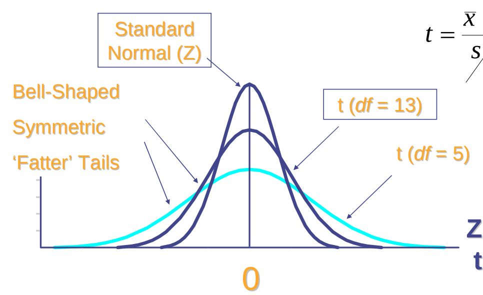
Student's t Distribution

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$



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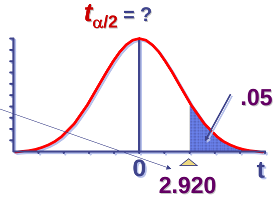
Student's t Table

df	Confidence interval	80%	90%	95%
	one tail	.10	.05	.025
	two tail	.20	.10	.05
1		3.078	6.314	12.706
2		1.886	2.920	4.303
3		1.638	2.353	3.182

For a 90% C.I.:
***n* = 3**
***df* = *n* - 1 = 2**
α = .10
α/2 =.05
$t_{\alpha/2}$ **= ?**



t values

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Estimation Example Mean (σ Unknown)

A random sample of weights of 25 subjects, has a sample mean 140 and sample standard deviation 8. Set up a 95% confidence interval estimate for μ.

$$1-\alpha = .95, \quad \alpha = 1-.95 = .05, \quad \alpha/2 = .025,$$

$$t_{\alpha/2,\, df=24} = t_{0.025} = 2.064$$

$$\bar{X} \pm t_{\alpha/2, n-1}\cdot\frac{S}{\sqrt{n}}$$

$$140 \pm 2.064\cdot\frac{8}{\sqrt{25}} \Rightarrow 140 \pm 3.31$$

$$\Rightarrow (136.69 \,,\, 143.31\;)$$

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Thinking Challenge

The numbers of community hospital beds per 1000 population that are available in each different regions of the country is normally distributed. A random sample 6 regions were selected and the rates of beds per 1000 were recorded and they are

3.6, 4.2, 4.0, 3.5, 3.8, 3.1.

Find the **90%** confidence interval estimate of the **mean** bed-rate in the country.

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Confidence Interval Solution*

$$\bar{X} \pm t_{\alpha/2, n-1} \cdot \frac{S}{\sqrt{n}}$$

$\bar{X} = 3.7$

$s = 0.38987$ $\quad n = 6, \quad \text{df} = n - 1 = 6 - 1 = 5$

$1-\alpha = .90, \quad \alpha = 1 - .90 = .1, \quad \alpha/2 = .05,$

$t_{\alpha/2, df=5} = t_{0.05} = 2.015$

$$3.7 \pm 2.015 \cdot \frac{.38987}{\sqrt{6}} \Rightarrow 3.7 \pm .321$$

$(3.379, 4.021)$

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Confidence interval with *z*-score:

The $(1-\alpha)\%$ confidence interval estimate for population mean:

- Assumption: If sampled from normal population with known variance, σ,

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

- Assumption: If large sample and if unknown variance, s replaces σ,

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

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Confidence interval with *t*-score:

The $(1-\alpha)\%$ confidence interval estimate for population mean:

Assumption: If sampled from normal population with unknown variance, σ,

$$\bar{x} \pm t_{\alpha/2, df=n-1} \cdot \frac{s}{\sqrt{n}}$$

(If sample size is large the normality assumption is insignificant.)

$t \to z$ *as sample becomes large*

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Average Weight for Female Ten Year Children In US

Info. from a random sample: $n = 10$, $\bar{x} = 80$ lb, $s = 18.05$ lb, assume weight is **normally distributed**, find the 95% confidence interval estimate for average weight.

Data:

73.80	50.00	101.40	67.20	102.20
97.80	81.00	93.40	63.20	70.00

How do we know whether normality assumption is OK?

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Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
ght (pounds) of particip	.171	10	.200*	.930	10	.452

. This is a lower bound of the true significance.

. Lilliefors Significance Correction

Both are greater than 0.05, normality assumption is acceptable.

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Average Weight for Female Ten Year Children In US

Info. from a random sample: n = 10, $\bar{x}$ = 80 lb, s = 18.05 lb, assume weight is normally distributed, find the 95% confidence interval estimate for average weight.

$$t_{\alpha/2} = t_{.05/2} = t_{0.025}, \quad \text{d.f.} = 10 - 1 = 9, \quad t_{0.025,\,9} = 2.262$$

$$\bar{x} \pm t_{\alpha/2, df=9} \cdot \frac{s}{\sqrt{n}} \Rightarrow 80 \pm 2.262 \cdot \frac{18.05}{\sqrt{10}}$$

$$80 \pm 12.91 \quad \Rightarrow (67.09,\ 92.91)$$

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Weight for Ten Year Old

Descriptives 80 ± 12.91

	What is your sex?			Statistic	Std. Error
nt (pounds) rticipant	female	Mean		80.0000	5.70840
		95% Confidence Interval for Mean	Lower Bound	67.0867	
			Upper Bound	92.9133	
		5% Trimmed Mean		80.4333	
		Median		77.4000	
		Variance		325.858	
		Std. Deviation		18.05153	
		Minimum		50.00	
		Maximum		102.20	
		Range		52.20	
		Interquartile Range		32.5000	
		Skewness		-.148	.687
		Kurtosis		-1.229	1.334
	male	Mean		86.8600	3.96048
		95% Confidence Interval for Mean	Lower Bound	77.9008	
			Upper Bound	95.8192	

Confidence Interval Estimate of Proportion

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Proportion Estimation

Parameter: Population Proportion p (or π)

(Percentage of people has no health insurance)

Statistic: Sample Proportion $\hat{p} = \frac{x}{n}$

x is number of successes

n is sample size

Remark: If data is coded as 1 or 0, sample mean is the same as sample proportion of 1's.

Data: 1, 0, 0, 1, 0 $\Rightarrow \bar{x} = \frac{1+0+0+1+0}{5} = \frac{2}{5} = .2 = p$

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Confidence Interval Proportion

1. Assumptions
   - Two Categorical Outcomes (Binomial Exp.)
   - Normal Approximation Can Be Used If
     - $n\hat{p} \pm 3\sqrt{n\hat{p}(1-\hat{p})}$ Does Not Include 0 or 1
     - or expect to see at least 5 counts in each category.
2. Confidence Interval Estimate (For large sample)

$$\left(\ \hat{p} - z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}},\ \hat{p} + z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}}\ \right)$$

$$1 - \hat{p} = \hat{q}$$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}}$$

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Estimation Example Proportion

A random sample of 400 from a large community showed that 32 have diabetes. Set up a 95% confidence interval estimate for p, the percentage of people that have diabetes.

$$\hat{p} = \frac{32}{400} = .08, \quad n = 400, \quad z_{\alpha/2} = z_{.025} = 1.96$$

$$\hat{p} \pm Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\cdot(1-\hat{p})}{n}} \qquad .08 \pm 1.96 \cdot \sqrt{\frac{.08\cdot(1-.08)}{400}}$$

$$.08 \pm .027 \ \Rightarrow 8\% \pm 2.7\%$$

$$\Rightarrow (5.3\%, 10.7\%)$$

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Thinking Challenge

A member of a health department wish to see what percentage of people in a community will support an environmental policy. Of **200** survey forms sent and received, **35** responded that they support the policy and the rest of them do not support the policy.

Find a **90%** confidence interval estimate of the percentage of the population in this community that support t**he policy**?

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Confidence Interval Solution*

$$\hat{p} = \frac{35}{200} = .175,\ n = 200,\ z_{\alpha/2} = 1.645$$

$$\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$$

$$.175 \pm 1.645 \cdot \sqrt{\frac{.175 \cdot (.825)}{200}}$$

$$.175 \pm .0442 = 17.5\% \pm 4.42\%$$

$$= (13.08\%,\ 21.92\%)$$

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Example:

Researchers wish to estimate the percentage of hospital employees infected by SARS in a certain country. Out of 500 randomly chosen hospital employees, 14 were infected. Find the 95% confidence interval estimate for percentage of hospital employees infected by SARS in this country.

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Sample Size

C.I. : $\hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$

Margin of Error $= B = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p} \cdot (1-\hat{p})}{n}}$

$n = \frac{z_{\alpha/2}^2}{B^2} \cdot \hat{p} \cdot (1-\hat{p})$ if pilot study is done.

or

$n = \frac{z_{\alpha/2}^2}{B^2} \cdot 0.25$ to get the largest sample to achieve the goal.

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Sample Size (No prior information on *p*)

Sample Size Example: If one wishes to do a survey to estimate the population proportion with 95% confidence and a margin of error of 3%, how large a sample is needed?

$Z_{\alpha/2} = 1.96$; $B = .03$

$n = (1.96^2/.03^2) \times .25 = 1067.11$

A sample of size 1068 is needed.

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Sample Size (With prior information on *p*)

Sample Size Example: If one wishes to to estimate the percentage of people infected with West Nile in a population with 95% confidence and a margin of error of 3%, how large a sample is needed? (A pilot study has been done, and the sample proportion was 6%.)

$Z_{\alpha/2} = 1.96$; $B = .03$

$n = (1.96^2/.03^2) \times .06 \times (1 - .06) = 240.7$

A sample of size 241 is needed.

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