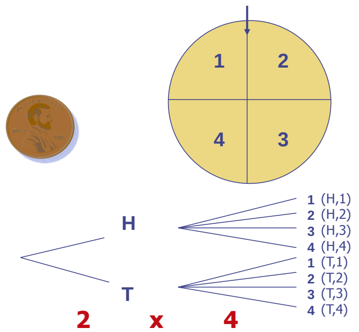
Sample Space = {(H,1),(H,2),(H,3),(H,4),(T,1),(T,2),(T,3),(T,4)}

Tree Diagram

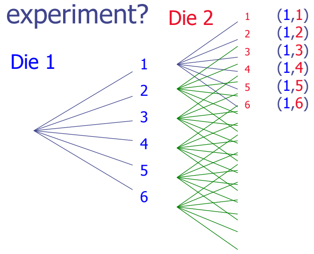


2 x 4

1

Tree Diagram

What is the sample space for casting two dice experiment?



6 x 6 = 36 outcomes

2

Sample Space (Two Dice)

Compound event

Sample Space:

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

Simple event

How many outcomes are "**Sum is 7**"?

3

Counting Rules

Fundamental Principle of Counting

Counting Rule: (use of the notation k_n)
In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total possibilities of the sequence will be $k_1 \times k_2 \times k_3 \times \ldots \times k_n$

2 x 4 = 8 possible outcomes (coin & wheel)
6 x 6 = 36 possible outcomes (2 dice)
$k_1 \times k_2$

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Example:

There are three shirts of different colors, two jackets of different styles and five pairs of pants in the closet. How many ways can you dress yourself with one shirt, one jacket and one pair of pants selected from the closet?

Sol: 3 x 2 x 5 = **30** ways
$k_1 \times k_2 \times k_3$

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Permutations

B A S E

How many different four-letter code words can be formed by using the four letters in the word "BASE" without repeating use of the same letter?

$k_1 = 4$ $k_2 = 3$ $k_3 = 2$ $k_4 = 1$

4·3·2·1 = **24**

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Factorial Formula

For any counting number n

n! = 1 x 2 x … x (n – 1) x n
0! = 1

Example: 3! = 1 x 2 x 3 = **6**

B A S E

How many different four-letter code words can be formed by using the four letters in the word "BASE" and the same letter can be used repeatedly?

$k_1 = 4$ $k_2 = 4$ $k_3 = 4$ $k_4 = 4$

4·4·4·4 = **256**

B A S E

How many different four-letter code words can be formed by using the four letters selected from letters A through Z and the same letter can **not** be used repeatedly?

$k_1 = 26$ $k_2 = 25$ $k_3 = 24$ $k_4 = 23$

26·25·24·23 = **358,800**

B A S E

How many different four-letter code words can be formed by using the four letters selected from letters A through Z and the same letter can be used repeatedly?

$k_1 = 26$ $k_2 = 26$ $k_3 = 26$ $k_4 = 26$

26·26·26·26 = **456,976**

Permutation Rule

Permutation Rule:

The number of possible permutations of r objects from a collection of n distinct objects is

$$_nP_r = \frac{n!}{(n-r)!}$$

Order does count!

Permutations

$$_nP_r = \frac{n!}{(n-r)!}$$

How many ways can a four-digit code be formed by selecting 4 distinct digits from nine digits, 1 through 9, without repeating use of the same digit?

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9\cdot8\cdot7\cdot6\cdot\cancel{5!}}{\cancel{5!}}$$

$= 9\cdot8\cdot7\cdot6 =$ **3024**

Combination Rule

Combination Rule:

The number of possible combinations of r objects from a collection of n distinct objects is

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

Order does not count!

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Combinations

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

How many ways can a committee be formed by selecting 3 people from a group 10 candidates?

n = 10, r = 3

$${}_{10}C_3 = \frac{10!}{3!\cdot(10-3)!} = \frac{10!}{3!\cdot 7!} = \frac{10\cdot 9\cdot 8\cdot \cancel{7!}}{3!\cdot \cancel{7!}}$$

$$= \frac{10\cdot 9\cdot 8}{3!} = \mathbf{120}$$

ABC, ACB, BAC, BCA, CAB, CBA are the same combination.

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Combinations

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

How many ways can a combination of 4 distinct digits be selected from nine digits, 1 through 9?

$${}_9C_4 = \frac{9!}{4!\cdot(9-4)!} = \frac{9!}{4!\cdot 5!} = \frac{9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{4!\cdot \cancel{5!}}$$

$$= \frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}$$

$$= 126$$

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Probability Using Counting (Theoretical Approach)

$${}_nC_r = \frac{n!}{r!\cdot(n-r)!}$$

In a lottery game, one selects 6 distinct numbers from a set of 47 distinct numbers, what is the probability of winning the jackpot?

n = 47, r = 6

$$= \frac{47!}{6!\cdot 41!} = \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot 42\cdot 41!}{6!\cdot 41!}$$

$$= \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot 42}{6!} = \mathbf{10,737,573}$$

$$P(\text{Jackpot}) = \frac{1}{10,737,573}$$ (Theoretical Prob.)

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Birthday Problem

In a group of randomly select 23 people, what is the probability that at least two people have the same birth date? (Assume there are 365 days in a year.)

P(at least two people have the same birth date)

Too hard !!!

= 1 – P(everybody has different birth date)

= 1 – [365x364x…x(365-23+1)] / 365^{23}

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Binomial Probability

What is the probability of getting two 6's in casting a balanced die 5 times experiment?

$P(S\cap S\cap S'\cap S'\cap S') = (1/6)^2 \times (5/6)^3 = \mathbf{0.016}$

$P(S\cap S'\cap S\cap S'\cap S') = (1/6)^2 \times (5/6)^3$

$P(S\cap S'\cap S'\cap S\cap S') = (1/6)^2 \times (5/6)^3$

…

How many of them? $\binom{5}{2} = \frac{5!}{2!\cdot 3!} = 10$

Probability (two 6's) = 0.016 x 10 = **0.16**

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