Goodness of Fit Test

The color distribution of A&A candies is supposed to be 30% red, 20% green, and 50% yellow. Does a randomly selected sample of A&A candies suggest that the distribution of different color of candies still follows the 30/20/50 distribution?

1

Chi-square Goodness of Fit Test
(For Distribution of A Variable)

The Chi-square Goodness-of-fit Test (Pearson, 1900)

H_0: The distribution of the population follow a specified distribution.

H_a: The distribution of the population does not follow a specified distribution.

	Classes				
	1	2	...	*k*	***Total***
Observed frequencies	O_1	O_2	...	O_k	***N***
A specified distribution	p_1^*	p_2^*		p_k^*	**1**
Expected frequencies	E_1	E_2	...	E_k	***N***

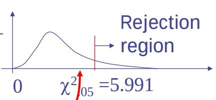
(Expected Cell Count: $E_j = \pi_j^* N$ $\quad j = 1, 2, ..., k$)

2

Example: (continue)

	Red	Green	Yellow	Total
Observed Frequencies: O_i	56	52	92	200
Expected Frequencies: E_i	60	40	100	200

$$\chi^2 = \frac{(56-60)^2}{60} + \frac{(52-40)^2}{40} + \frac{(92-100)^2}{100}$$



$= 4.507 < 5.991 = \chi^2_{.05}(2)$

=> fail to reject H_0

Conclusion: There is no sufficient evidence to support that the color distribution is significantly different from 30/20/50.

3

Test Statistic:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2\{k-1\}$$

Under H_0, χ^2 has a chi-square distribution with $(k-1)$ degrees of freedom

Cochran's guidelines: (Large sample conditions)

- None of the expected cell counts less than 1.
- No more than 20% of the expected cell frequencies are less than 5.

Decision Rule:

Reject H_0 If $\chi^2 > \chi^2_\alpha$ or ***p*-value** $< \alpha$.

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Example:

It is believe that the outcomes is uniformly distributed from a game of drawing a ball from 4 balls numbered from 1, 2, 3, and 4 in a box. The outcomes from the past 100 games were recorded, and the frequency distribution (count) is the following: 1 occurred 20 times, 2 occurred 25 times, 3 occurred 41 times and 4 occurred 14 times. Does the sample suggest that the outcome distribution of this game is not significantly different from uniform distribution?

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Chi-square Test for Independence

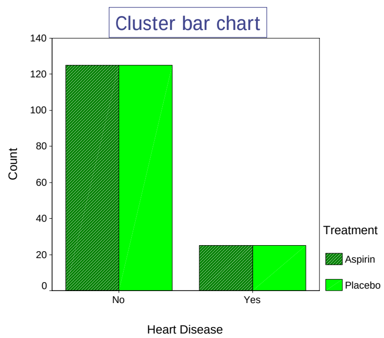
Statistical Correlation Between **Categorical** Variables

Is there a relationship between **Treatment** and **Heart Disease?**

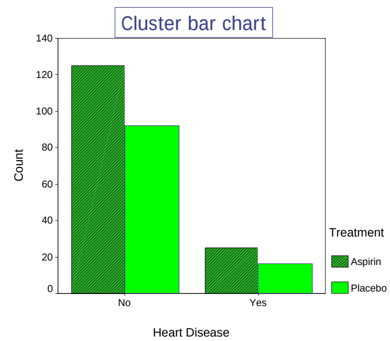
Heart Disease:
"Have the disease" or "Do not have the disease."
Treatment:
"Placebo" or "Aspirin".

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36	114	150
Aspirin	14	136	150
Total	50	250	300

Evidence of No Correlation

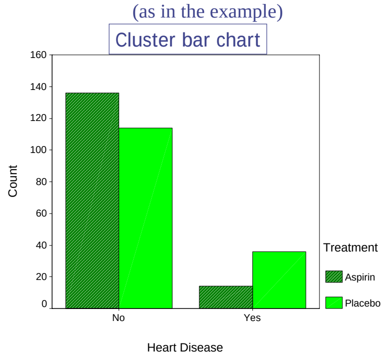


Evidence of No Correlation



Evidence of Correlation

(as in the example)



Questions about the sample

- Did this happen by chance?
- How likely would it happen if there is no significant treatment effect?

Testing statistical hypotheses!

Chi-square Test for Independence

Null hypothesis: There is **no relationship** between treatment variable and outcome variables.

Alternative hypothesis: There **is relationship** between treatment variable and outcome variables

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36 (25)	114 (125)	150
Aspirin	14 (25)	136 (125)	150
Total	50	250	300

$\frac{150 \times 50}{300} = 25$

Observed frequency Expected frequency

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Expected Frequency:

Numbers in (..) :
(i,j)th cell expected freq. = $\frac{M_i \times N_j}{T}$

M_i: i-th **row** total
N_j: j-th **column** total
T : grand total

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Chi-square Test for Independence

Test statistic:

χ^2 = $(36 - 25)^2/25 + (14 - 25)^2/25 + (114 - 125)^2/125 + (136 - 125)^2/125 =$ **11.616**

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36 (25)	114 (125)	150
Aspirin	14 (25)	136 (125)	150
Total	50	250	300

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Test Statistic:

$$\chi^2 = \sum_{i=1}^{rc} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2\{(r-1)(c-1)\}$$

Chi-square distribution with (r–1)(c–1) degrees of freedom

Cochran's guidelines: (Assumption: Large sample.)

- None of the expected cell counts less than 1.
- No more than 20% of the expected cell frequencies are less than 5.

Decision Rule:

If $\chi^2 > \chi^2_\alpha$ or *p*-value < α, the null hypothesis is rejected.

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Decision Rule

- *p*-value approach:
  If the *p*-value is less than the level of significance for the test (usually at .05 or 5%), the null hypothesis would be rejected. This means, statistically, there is sufficient evidence to support the alternative hypothesis.
- Critical Value approach:
  If the test statistic value is greater than the Critical Value (is $\chi^2_\alpha = 3.841$ at 5% level of significance), the null hypothesis is rejected.

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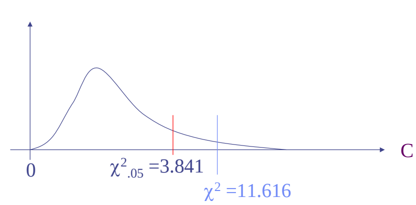
Critical value

If the test statistic value is greater than the Critical Value (is $\chi^2_\alpha = 3.841$ at 5% level of significance), the null hypothesis is rejected.



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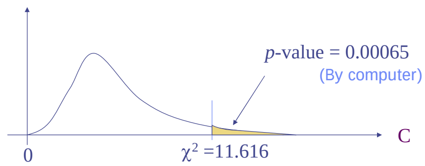
p-value

The probability of observing an evidence (statistic) that is at least as extreme as the observed sample evidence, if the null hypothesis is true.



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Decision (Conclusion)

- *p*-value approach:

  If the *p*-value = **0.00065** is less than the level of significance for the test (usually at .05 or 5%), the null hypothesis would be rejected. This means, statistically, there is sufficient evidence to support the alternative hypothesis.

- Critical Value approach:

  Critical Value is $\chi^2_{\alpha} = 3.841$ at 5% level of significance. Since $\chi^2 =$ **11.616 > 3.841**, the null hypothesis is rejected.

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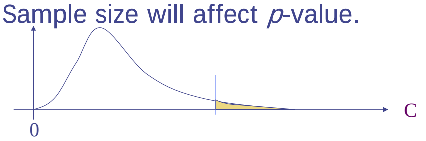
p-value

- The smaller the *p*-value the stronger the evidence for rejecting the null hypothesis and supporting the alternative hypothesis.
- The interpretation is similar for all statistical testing.
- Sample size will affect *p*-value.



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Relative Risk

Relative risk of getting heart disease for taking Placebo versus Aspirin is $\frac{36/150}{14/150} = 2.57$

	Heart Disease		
Treatment	Yes +	No –	Total
Placebo	36	114	150
Aspirin	14	136	150
Total	50	250	300

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Cautions

- Confounding factors: race, age, ...
- Randomization, Design of experiment.
- Use of regression or multiple contingency tables.

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Step 1: Test the hypothesis at 5% level of sig.

Hypothesis:

H_0: There is **NO** relation between variable 1 (treatment) and variable 2 (outcome variables).

H_a: There is relation between two variables.

	Relapse		
	No	Yes	Total
Desipramine	14 (8)	10 (16)	24
Lithium	6 (8)	18 (16)	24
Placebo	4 (8)	20 (16)	24
Total	24	48	72

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Step 2:

Test Statistics: $\chi^2 = \frac{(14-8)^2}{8} + \frac{(10-16)^2}{16} + \frac{(6-8)^2}{8} + \frac{(18-16)^2}{16} + \frac{(4-8)^2}{8} + \frac{(20-16)^2}{16} = \mathbf{10.5}$

Step 3:

d.f.= (3–1)(2–1) = 2

C.V. approach: If $\chi^2 > \chi^2_{.05} = 5.99$ (c.v.) , reject null hypothesis.

p-value approach: If the ***p*-value** of the test is less than 0.05, null hypothesis is rejected.

Step 4:

Conclusion: Since $\chi^2 = 10.5 > \chi^2_{.05} = 5.99$ or $\chi^2 = 10.5 > 9.210$, the ***p*-value** of the test is less than 0.05. The relation between treatment and outcome variables is statistically significant.

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Example:

To test whether the percentages of people who prefer to drink water than coke are the same for people living in city A and city B, two random samples were taken from both cities and the results are in the following table.
At $\alpha = 0.05$, can you conclude that the percentages of people who prefer to drink water than coke in these two cities are different?

	Water	Coke
City A	41	19
City B	12	28

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